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ENTITLED

PROCESS AND APPARATUS FOR FAST ASSIGNMENT
OF OBJECTS TO A RECTANGLE

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PROCESS AND APPARATUS FOR FAST ASSIGNMENT OF OBJECTS TO A RECTANGLE

FIELD OF THE INVENTION

This invention concerns placement of objects
5 in a rectangle, and particularly to placement of cells
in integrated circuit designs during creation of a
floorplan.

BACKGROUND OF THE INVENTION

The present invention is directed to a
10 process and apparatus for assigning N objects to M
points of a rectangle, where $M \geq N$. The invention is
particularly useful in placement of cells in an
integrated circuit (IC) chip, although it is also
useful in other environments where a large number of
15 object must be placed in a space.

Consider a rectangle having left and right
edges a and b , where $a < b$, and bottom and top edges
 c and d , where $c < d$. The rectangle containing a
point having coordinates (x, y) can be defined as
20 $R = \{(x, y) | a \leq x \leq b, c \leq y \leq d\}$.

Points $P_1(x_1, y_1), P_2(x_2, y_2), \dots, P_M(x_M, y_M)$, are
the points of rectangle R . Objects $Q_1(x'_1, y'_1),$
 $Q_2(x'_2, y'_2), \dots, Q_N(x'_N, y'_N)$ are the objects to be
placed in rectangle R . T is a set of types of points
25 and objects. t_i is a type of a point P_i , where i is
each member of the sequence of 1 to M ($i = \overline{1, M}$).
Thus, $t_i \in T$. t'_i is a type of an object $Q_i, i = \overline{1, N}$,
thus $t'_i \in T$. For any pair of types, $u \in T$ and $v \in T$,

the relation of these types $TR(u,v)$ is such that if the object having a type v is allowed to be assigned to a point of type u , $TR(u,v) = 1$. Otherwise, $TR(u,v) = 0$. $TR(u,u) = 1$ for each type $u \in T$.

5 The goal of objects assignment is to assign all objects $Q_i, i = \overline{1, N}$ to points $P_{s(i)}$ (or to find the assignment $s(i)$ for each $i = \overline{1, N}$), so that:

- a) $s(i_1) \neq s(i_2)$ for any $1 \leq i_1 < i_2 \leq N$,
- b) $TR(t_{s(i)}, t'_i) = 1$ for any $i = \overline{1, N}$, and
- 10 c) distances between objects Q_i and points $P_{s(i)}$ is as small as possible.

The cost $C(i,j)$ of assignment of an object Q_i to a point P_j is denoted as follows:

$$C(i,j) = \begin{cases} (x_j - x'_i)^2 + (y_j - y'_i)^2 & \text{if } TR(t_j, t'_i) = 1 \\ \infty & \text{if } TR(t_j, t'_i) = 0 \end{cases}$$

15 One well-known technique of assigning objects is Kuhn's algorithm, which finds the optimal solution $s(i)$ such that the sum $\sum_{i=1}^N C(i, s(i))$ is the smallest possible value. However, Kuhn's algorithm requires a considerable amount of time to execute.

20 More particularly, execution of Kuhn's algorithm requires time defined as $O(N^2 M^2)$. In practice, cell placement might require processing cell assignments for large values of parameters N and M ($N, M > 1000$), and might be repeated many times during the design

25 process. Consequently, execution of Kuhn's algorithm requires an unacceptable amount of time. Therefore,

a need exists for a technique to quickly estimate objects assignment.

SUMMARY OF THE INVENTION

In a first embodiment, a process assigns
5 objects to points of a first rectangle. An initial
assignment of the objects to points of the first
rectangle is created. The first rectangle is divided
into a plurality of second, smaller rectangles. An
object assignment procedure is applied to the
10 initially assigned objects in each second rectangle.
Preferably, each point in the first rectangle is in
at least two second rectangles.

In some embodiments, the initial assignment
of objects is performed by calculating a maximal cost
15 of assignment of object to points, and selecting an
assignment of objects having a minimum value of
maximal cost. More particularly, a maximal matching
assignment is calculated by iteratively recalculating
the maximal matching assignment based on a midpoint of
20 between the minimum and maximum costs of the assignment
calculated during the prior iteration, and then
recalculating the minimum and maximum costs of the
recalculated assignment, iterating until the minimum
cost is not smaller than the maximum cost.

25 In other embodiments, the object assignment
procedure comprises finding objects assigned to
points of each second rectangle, applying Kuhn's
algorithm of object assignment to the second
rectangle, and correcting the assignment. This

procedure is iteratively repeated until the assignment does not change.

In accordance with another embodiment of the present invention, the process is carried out in
5 a computer under the control of a computer readable program having computer readable code that causes the computer to carry out the process.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. 1 is a flowchart of a process of
10 assigning objects in accordance with an embodiment of the present invention.

FIG. 2 is a flowchart of initial object assignment used in the process of FIG. 1.

FIG. 3 is a diagram useful in explaining
15 the process of FIG. 1.

Fig. 4 is a flowchart of application of Kuhn's algorithm used in the process of FIG. 1.

DETAILED DESCRIPTION OF THE PREFERRED EMBODIMENTS

FIG. 1 is a flowchart of a process of
20 object assignment in accordance with a presently preferred embodiment of the present invention. The process is preferably carried out by a computer performing the process under control of a computer readable program code. The process commences at step
25 10 by evaluating the costs $C(i,j)$ of assignment of objects Q_i to points P_j , where $i=\overline{1,N}$ and $j=\overline{1,M}$. The costs are evaluated in accordance with the definition of costs assigned by the designer. At step 12, an initial assignment $s(i)$ of objects Q_i to points P_j is

created to achieve a minimization of the maximal cost. The process of initial assignment is more fully described in connection with FIG. 2.

Step 12, shown in detail in FIG. 2, obtains an initial assignment $s(i)$ that results from the minimization of the maximal value of the cost: $\max_{1 \leq i \leq N} (C(i, s(i)))$. In order to simplify this step the process employs rounded costs $\bar{C}(i, j)$ instead of the original costs $C(i, j)$, where

$$\bar{C}(i, j) = \begin{cases} \lfloor C(i, j) \rfloor & \text{if } C(i, j) \neq \infty \\ \infty & \text{if } C(i, j) = \infty \end{cases}$$

and where $\lfloor C(i, j) \rfloor$ is the maximal integer not greater than $C(i, j)$. For purposes of explanation, the maximal cost is designated C , $C = \max_{\bar{C}(i, j) \neq \infty} (\bar{C}(i, j))$.

At step 30, the terms LowerBound is set to 0, UpperBound is set to the maximal cost integer (C), and middle is set to the midvalue between LowerBound and UpperBound:

$$\begin{aligned} \text{LowerBound} &= 0 \\ \text{UpperBound} &= C \\ \text{middle} &= \left\lfloor \frac{\text{LowerBound} + \text{UpperBound}}{2} \right\rfloor. \end{aligned}$$

At step 32, the maximal matching assignment $s(i)$ is found for matching matrix

$$d(i, j) = \begin{cases} 1 & \text{if } \bar{C}(i, j) \leq \text{middle} \\ 0 & \text{if } \bar{C}(i, j) > \text{middle} \end{cases}$$

At step 34, if the maximal matching $s(i)$ obtained at step 32 is determined for each $i = \overline{1, N}$, such that the

maximal cost is not greater than the middle (maximal cost \leq middle), UpperBound is assigned to the value of middle, UpperBound = middle. Otherwise, LowerBound is assigned to the value of middle plus 1,
5 LowerBound = middle+1.

If, at step 36, LowerBound < UpperBound, the process returns to step 32. Otherwise at step 38, the initial assignment is the maximal matching $s(i)$ found at step 32 for the value
10 middle=LowerBound. The assignment thus obtained is output to step 14 (FIG. 1).

The process of step 12 calculates a maximal matching assignment by iteratively recalculating the maximal matching assignment based on a midpoint of
15 between the minimum and maximum costs of the assignment calculated during the prior iteration, and then recalculating the minimum and maximum costs of the recalculated assignment. The process continues to iterate until the minimum cost is not smaller than the
20 maximum cost (i.e., LowerBound \geq UpperBound).

The technique of finding the maximal matching $s(i)$ in the bipartitional graph determined by the matrix $d(i,j)$ is well-known. It is also well known that the time required for the solution of this
25 problem is $O(MN)$. The process of FIG. 2 is a binary-searching algorithm for finding the value $\max_{1 \leq i \leq N} (\bar{C}(i, s(i)))$. The LowerBound and UpperBound used in the algorithm are the lower and upper bounds of the

value $\max_{1 \leq i \leq N} (\bar{C}(i, s(i)))$. The number of repetitions of the steps 32-36 is $\lceil \log_2 C \rceil$, where $\lceil \log_2 C \rceil$ is the minimal integer number that is not less than $\log_2 C$. Consequently, time required to obtain the initial
5 assignment $s(i)$ is $O(\log_2 C \cdot MN)$.

As shown in FIG. 1, at step 14 a rectangle R is split into a set of small rectangles R_1, R_2, \dots, R_k . The splitting of rectangle R can be made by any of several techniques. The small rectangles $R_1,$
10 R_2, \dots, R_k has intersections such that every point of the rectangle R belongs to at least 2 small rectangles. FIG. 3 illustrates splitting a rectangle R into $k=41$ small rectangles, shown as 16 equal small rectangles in the upper portion of FIG. 3 and 25 more
15 small portions in the lower portion of the figure. Note that the small rectangles may have different sizes.

If M_q is the number of points, $P_j, j=1, \overline{M}$, that belong to a small rectangle R_q , it is evident
20 that more powerful splitting is obtained as the number of points of rectangle R_q becomes smaller (value of $\max_{1 \leq q \leq k} (M_q)$ becomes smaller). If rectangle R is uniformly split, then $M_q = O\left(\frac{M}{k}\right)$.

At step 16, each small rectangle $R_1, R_2, \dots,$
25 R_k , is examined using Kuhn's algorithm of assignment to correct the assignment $s(i)$. The procedure of step 16 is more fully described in connection with

FIG. 4. For each small rectangle R_q at step 40, all points $P_{j_1}, P_{j_2}, \dots, P_{j_{M_q}}$ that belong to the small rectangle R_q are considered. At step 42, all objects $Q_{i_1}, Q_{i_2}, \dots, Q_{i_{N_q}}$ that are assigned to points $P_{j_1}, P_{j_2}, \dots, P_{j_{M_q}}$ are located. Objects $Q_{i_1}, Q_{i_2}, \dots, Q_{i_{N_q}}$ are those objects Q_i for which $s(i) \in \{j_1, j_2, \dots, j_{M_q}\}$. At step 44, Kuhn's algorithm of object assignment is applied to place objects $Q_{i_1}, Q_{i_2}, \dots, Q_{i_{N_q}}$ at points $P_{j_1}, P_{j_2}, \dots, P_{j_{M_q}}$ and obtain an assignment $s'(i)$. At step 46, the assignment achieved at step 12 (FIG. 12) is corrected as

$$s(i_1) = s'(i_1), s(i_2) = s'(i_2), \dots, s(i_{N_q}) = s'(i_{N_q}).$$

The time required to perform Kuhn's algorithm to each small rectangle R_1, R_2, \dots, R_k at step 44 is $O(N_q^2 \cdot M_q^2) = O\left(\frac{M^4}{k^4}\right)$, so the time required to perform Kuhn's algorithm to all k small rectangles is $O\left(\frac{M^4}{k^3}\right)$, which is significantly shorter than the time required to apply Kuhn's algorithm of assignment to the entire rectangle R .

The process of FIG. 1 then continues to step 18 to determine whether the assignment $s(i)$ was changed at step 16. If assignment $s(i)$ did not change at step 16, the process continues to step 20 where the final assignment $s(i)$ is output. If $s(i)$ changed, the process returns to step 16 to re-define small rectangles and re-calculate Kuhn's algorithm.

Each iteration uses the assignment calculated in the prior iteration, with the iterations continuing until $s(i)$ does not change at step 18. Hence, each iteration produces a more accurate object assignment
5 than the prior iteration. As an alternative to detecting an unchanging assignment at step 18, the process could continue to step 20 upon some other convenient event, such as execution of a predetermined number of iterations of the loop
10 including step 16, or if the assignment change by the latest iteration of step 16 is less than some predetermined amount.

The present invention thus provides a good estimate of the placement of objects in a rectangle,
15 such as cells in the floorplan of an integrated circuit, in a shorter period of time than required by the Kuhn's algorithm. More particularly the time of performing the process of the present invention in a computer is the sum of the initial assignment (step
20 12) and the final assignment (step 16) and is

$$O(MN \cdot \log_2 C) + O\left(\frac{M^4}{k^3}\right).$$

Although the present invention has been described with reference to preferred embodiments, workers skilled in the art will recognize that
25 changes may be made in form and detail without departing from the spirit and scope of the invention.